

The Schrödinger equation is the fundamental governing equation of non-relativistic quantum mechanics. It plays the same role in the quantum world that Newton's second law ($F = ma$) plays in classical mechanics. Instead of predicting a precise trajectory, it calculates the wavefunction, which contains all the statistical information about a system.

Here is the mathematical breakdown of its two main forms.

1. The Time-Dependent Schrödinger Equation (TDSE)

This is the most general form of the equation. It describes how a quantum system evolves over time.

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H} \Psi(\mathbf{r}, t)$$

Breaking Down the Components:

i : The imaginary unit ($\sqrt{-1}$). Its presence ensures that quantum states wave and oscillate rather than simply decaying like heat.

\hbar : The reduced Planck constant ($\hbar = h / (2\pi)$), which sets the scale of the quantum world.

$(\partial) / (\partial t)$: The partial derivative with respect to time, representing how the system changes second by second.

$\Psi(\mathbf{r}, t)$: The wavefunction. It is a function of position \mathbf{r} and time t . The absolute square $|\Psi|^2$ gives the probability density of finding a particle at a specific place and time.

\hat{H} : The Hamiltonian operator. In quantum mechanics, physical observables are represented by operators. The Hamiltonian represents the total energy of the system.

2. The Hamiltonian Operator in Detail

To do actual calculations for a particle moving in space, we must expand the Hamiltonian operator \hat{H} into its kinetic and potential energy components:

$$\hat{H} = \hat{T} + \hat{V}$$

Where:

Kinetic Energy Operator (\hat{T}): Derived from momentum, it is expressed as $-\frac{\hbar^2}{2m}\nabla^2$, where m is the mass and ∇^2 (del-squared) is the Laplacian operator checking spatial curvature.

Potential Energy Operator (\hat{V}): Represented by $V(\mathbf{r}, t)$, which depends on the environment (e.g., an electric field or a gravitational well).

Putting it all together in 3D space:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t)$$

3. The Time-Independent Schrödinger Equation (TISE)

If the potential energy V does not depend on time ($V(\mathbf{r}, t) = V(\mathbf{r})$), we can use a mathematical technique called separation of variables. We split the wavefunction into a spatial part and a temporal part: $\Psi(\mathbf{r}, t) = \psi(\mathbf{r})e^{-iEt/\hbar}$.

Plugging this back in yields the Time-Independent Schrödinger Equation:

$$\hat{H}\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Or explicitly:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Mathematical Significance of the TISE:

An Eigenvalue Problem: Mathematically, this is an eigenvalue equation. \hat{H} is a linear operator, $\psi(\mathbf{r})$ is the eigenfunction, and E is the eigenvalue.

Quantization: Solving this equation under specific boundary conditions restricts the allowable values of E . This is precisely where "quantization" comes from in quantum mechanics—energy levels cannot be just anything; they must be specific discrete values.

Summary of Mathematical Interpretations

Concept	Mathematical Representation	Meaning
Probability Conservation	$\int_{-\infty}^{\infty} \Psi(\mathbf{r}, t) ^2 d\mathbf{r} = 1$	
Linearity	If ψ_1 and ψ_2 are solutions, then $c_1\psi_1 + c_2\psi_2$ is also a solution.	Explains quantum superposition.
Hermitian Operator	$\hat{H} = \hat{H}^\dagger$	Guarantees that the energy eigenvalues (E) will always be real numbers, which can be measured in a lab.

Document information

The Schrödinger equation

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