Chapter 3: Composite Systems, Entanglement, and Correlations

Quantum energy teleportation is not a one-particle story. It is a story about a composite quantum system: a system made from parts.

Alice acts on one part. Bob acts on another part. The many-body system connecting them may be a spin chain, a set of coupled oscillators, or a quantum field. The key resource is not that Alice sends a quantum particle to Bob. The key resource is that the joint quantum state can contain correlations between distant regions. Some of these correlations are classical in spirit, like two coins that were prepared to match. Others are genuinely quantum. The genuinely quantum kind is called entanglement.

In QET, entanglement and correlations in the ground state allow Alice's local measurement outcome to carry useful information about fluctuations near Bob. Bob cannot use that information until Alice sends it through an ordinary classical channel. This is why QET can be nonclassical without allowing faster-than-light signaling.

To understand that statement, we need the mathematics of composite quantum systems: tensor products, reduced density matrices, partial traces, separable and entangled states, Bell-type correlations, and mutual information. These ideas are standard in quantum information theory, and we will use the usual finite-dimensional formalism developed in texts such as Nielsen and Chuang (2010).

3.1 Why Composite Systems Need a New Mathematical Rule

Suppose Alice has one qubit and Bob has one qubit.

Alice's qubit has a two-dimensional Hilbert space, which we call

$$\mathcal{H}_A.$$

Bob's qubit also has a two-dimensional Hilbert space,

$$\mathcal{H}_B.$$

If we were describing two classical bits, we could simply list four possibilities:

$$00, \quad 01, \quad 10, \quad 11.$$

Quantum mechanics also needs four basic joint possibilities, but it allows superpositions of them. The mathematical structure that combines the two Hilbert spaces is called the tensor product.

The joint Hilbert space is

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B.$$

The symbol ⊗ means "combine these systems into one larger quantum system."

For two qubits, a standard basis of the joint Hilbert space is

$$|0\rangle_A \otimes |0\rangle_B, \quad |0\rangle_A \otimes |1\rangle_B, \quad |1\rangle_A \otimes |0\rangle_B, \quad |1\rangle_A \otimes |1\rangle_B.$$

These are usually written more compactly as

$$|00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle.$$

A general pure state of two qubits has the form

$$|\psi\rangle_{AB} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle,$$

where α,β,γ,δ∈ mathbbC and normalization requires

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1.$$

This is already richer than two classical bits. The amplitudes are complex numbers, and the state can be a superposition of different joint possibilities.

3.2 Product States: When the Parts Have Their Own States

The simplest composite quantum states are product states.

A product state is a state that can be written as

$$|\psi\rangle_{AB} = |\phi\rangle_A \otimes |\chi\rangle_B.$$

This means Alice's system has its own state $|\varphi\rangle_{(A)}$, Bob's system has its own state $|\chi\rangle_{(B)}$, and the joint state is just the combination of those two independent descriptions.

For example, suppose

$$|\phi\rangle_A = \frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}},$$

and

$$|\chi\rangle_B = |0\rangle_B.$$

Then

$$|\psi\rangle_{AB} = \left(\frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}}\right) \otimes |0\rangle_B.$$

Using linearity,

$$|\psi\rangle_{AB} = \frac{|00\rangle + |10\rangle}{\sqrt{2}}.$$

This is a superposition, but it is still a product state. Alice's qubit is in a superposition, Bob's qubit is in $|0\rangle$, and there is no special correlation between them.

A useful warning is this:

Not every superposition is entangled.

The state

$$\frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

is a superposition, but it factors as

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle.$$

So it is not entangled.

3.3 Entangled Pure States

A pure state of two systems is called entangled if it cannot be written as a product state.

The most famous examples are the Bell states. One of them is

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

This state cannot be written as

$$|\phi\rangle_A \otimes |\chi\rangle_B.$$

Let us see why.

Suppose it could. Write

$$|\phi\rangle_A = a|0\rangle + b|1\rangle,$$

and

$$|\chi\rangle_B = c|0\rangle + d|1\rangle.$$

Then

$$|\phi\rangle_A \otimes |\chi\rangle_B = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle.$$

To equal

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}},$$

we would need

$$ac = \frac{1}{\sqrt{2}}, \quad bd = \frac{1}{\sqrt{2}},$$

but also

$$ad = 0, \quad bc = 0.$$

The first two equations require a,c,b,d to be nonzero. But then ad and bc cannot both be zero. This is impossible. Therefore $|\Phi^+\rangle$ is entangled.

Entanglement means that the whole state contains information that cannot be assigned separately to the parts. The joint state is definite, but the individual subsystems do not have pure states of their own.

This feature is central to quantum information theory and is treated as one of the distinguishing resources separating quantum systems from classical ones (Horodecki et al., 2009).

3.4 What Entanglement Looks Like in Measurements

Consider the Bell state

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

If Alice measures her qubit in the computational basis \|0⟩,|1⟩\, she obtains outcome 0 with probability 1/2 and outcome 1 with probability 1/2.

If Alice obtains 0, Bob's qubit is found in $|0\rangle$.

If Alice obtains 1, Bob's qubit is found in $|1\rangle$.

So Alice and Bob's outcomes are perfectly correlated in this basis.

But this does not mean Alice controlled Bob's result. Alice's outcome was random. She did not choose whether she obtained 0 or 1. What she gained was information about Bob's correlated outcome.

This distinction is essential for QET. In QET, Alice's measurement gives her information about distant quantum degrees of freedom. But the outcome is random, and Bob cannot know which operation to perform until Alice sends him the outcome through a classical channel.

3.5 Density Matrices for Composite Systems

In Chapter 2, we introduced density matrices. For a pure state $|\psi\rangle$, the density matrix is

$$\rho = |\psi\rangle\langle\psi|.$$

For the Bell state,

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}},$$

the joint density matrix is

$$\rho_{AB} = |\Phi^+\rangle\langle\Phi^+|.$$

Expanding,

$$\rho_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right).$$

The terms

$$|00\rangle\langle 00| \quad \text{and} \quad |11\rangle\langle 11|$$

look like ordinary alternatives. The terms

$$|00\rangle\langle 11| \quad \text{and} \quad |11\rangle\langle 00|$$

are coherence terms. They contain phase-sensitive quantum information about the superposition.

Density matrices are especially useful because QET deals with local measurements, conditional operations, and subsystems. To describe what Alice alone or Bob alone can observe, we need the reduced density matrix.

3.6 Reduced Density Matrices: The State Seen Locally

Suppose we know the full state $\rho_{(AB)}$ of Alice and Bob together. What state should Bob assign to his subsystem alone?

The answer is the reduced density matrix.

Bob's reduced density matrix is written

$$\rho_B = \mathrm{Tr}_A(\rho_{AB}).$$

This operation is called the partial trace over Alice's system.

The idea is simple: if Bob has access only to system B, then all predictions for Bob's local measurements must be computed from $\rho_{(B)}$. Alice's degrees of freedom are mathematically "summed over."

Similarly,

$$\rho_A = \mathrm{Tr}_B(\rho_{AB})$$

is Alice's reduced density matrix.

The partial trace is defined by the requirement that for every Bob-only observable $O_{(B)}$,

$$\mathrm{Tr}_{AB}[\rho_{AB}(I_A \otimes O_B)] = \mathrm{Tr}_B(\rho_B O_B).$$

In words: using the full state and a Bob-local observable must give the same expectation value as using Bob's reduced state alone.

This is not just a convenient definition. It is what makes reduced density matrices physically meaningful.

3.7 Example: Reduced State of a Bell Pair

Let

$$\rho_{AB} = |\Phi^+\rangle\langle\Phi^+|.$$

We calculate

$$\rho_B = \mathrm{Tr}_A(\rho_{AB}).$$

Using

$$\rho_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right),$$

trace over Alice's basis \|0⟩₍A₎,|1⟩₍A₎\.

The rule is

$$\mathrm{Tr}_A\left(|i\rangle_A|j\rangle_B\langle k|_A\langle l|_B\right) = \langle k|i\rangle\, |j\rangle_B\langle l|_B.$$

Since

$$\langle k|i\rangle = \delta_{ki},$$

terms survive only when Alice's bra and ket match.

Now apply this to each term:

$$\mathrm{Tr}_A(|00\rangle\langle 00|) = |0\rangle_B\langle 0|,$$

$$\mathrm{Tr}_A(|00\rangle\langle 11|) = 0,$$

$$\mathrm{Tr}_A(|11\rangle\langle 00|) = 0,$$

$$\mathrm{Tr}_A(|11\rangle\langle 11|) = |1\rangle_B\langle 1|.$$

Therefore

$$\rho_B = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right).$$

So

$$\rho_B = \frac{I}{2}.$$

Bob's local state is maximally mixed. He has no definite pure state. If he measures in the computational basis, he gets 0 or 1 with equal probability. If he measures in any other qubit basis, he also sees a maximally random state.

This is a deep feature:

The whole Bell pair is in a pure state, but each part alone is in a mixed state.

That cannot happen for a product pure state. It is a signature of entanglement.

3.8 Classical Correlation Versus Quantum Entanglement

Entanglement is not the same as ordinary correlation.

Consider the mixed state

$$\rho_{AB}^{\mathrm{classical}} = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|.$$

This state describes a pair of qubits that are classically correlated in the computational basis. If Alice measures and obtains 0, Bob will obtain 0. If Alice obtains 1, Bob will obtain 1.

But this is not the Bell state.

The Bell state density matrix is

$$\rho_{AB}^{\text{Bell}} = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right).$$

The difference is the presence of the off-diagonal coherence terms

$$|00\rangle\langle 11| \quad \text{and} \quad |11\rangle\langle 00|.$$

The classical correlated state contains uncertainty about which product state was prepared. The Bell state contains a coherent superposition of the two joint alternatives.

This distinction matters physically. If Alice and Bob both measure in the computational basis, the two states give the same statistics. But if they measure in other bases, the Bell state shows correlations that the classical mixture does not.

For example, define

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

The Bell state can also be written as

$$|\Phi^+\rangle = \frac{|++\rangle + |--\rangle}{\sqrt{2}}.$$

So it is perfectly correlated in the \|+⟩,|-⟩\ basis too.

The classical mixture

$$\frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$$

does not have the same perfect correlation in the \|+⟩,|-⟩\ basis. The coherence terms are missing.

This is one way entanglement reveals itself: not merely through correlation in one measurement basis, but through a pattern of correlations across incompatible measurement choices.

3.9 Separable and Entangled Mixed States

For pure states, the definition of entanglement is simple: a pure state is entangled if it cannot be written as a product state.

For mixed states, the definition is slightly more subtle.

A bipartite mixed state $\rho_{(AB)}$ is called separable if it can be written as

$$\rho_{AB} = \sum_k p_k \, \rho_A^{(k)} \otimes \rho_B^{(k)},$$

where

$$p_k \geq 0, \quad \sum_k p_k = 1.$$

This means the state can be understood as a probabilistic mixture of product states. The correlations may be strong, but they can be explained by shared classical randomness.

For example,

$$\rho_{AB}^{\text{classical}} = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$$

is separable because it is already written as

$$\frac{1}{2}\left(|0\rangle\langle 0|\right) \otimes \left(|0\rangle\langle 0|\right) + \frac{1}{2}\left(|1\rangle\langle 1|\right) \otimes \left(|1\rangle\langle 1|\right).$$

A mixed state is entangled if no such decomposition exists.

This definition is operationally important: separable states can be prepared using local operations and shared classical randomness, while entangled states cannot generally be prepared that way. The structure and detection of mixed-state entanglement is a major topic in quantum information theory (Horodecki et al., 2009).

In QET, the relevant states are often ground states of interacting many-body systems. These are typically not simple Bell pairs. They may contain complicated patterns of entanglement and correlation across many sites or spatial regions. But the basic idea is the same: local regions may be correlated in ways that cannot be reduced to independent local states.

3.10 Local Observables in Composite Systems

An observable is represented by an operator. In a composite system, an observable may act only on Alice’s subsystem, only on Bob’s subsystem, or on both.

If M_A is an observable on Alice’s Hilbert space $\mathcal{H}_A$, then the corresponding observable on the full system is

$$M_A \otimes I_B.$$

The identity operator I_B means “do nothing to Bob’s subsystem.”

Similarly, a Bob-local observable is

$$I_A \otimes N_B.$$

A joint observable might be

$$M_A \otimes N_B.$$

For a state ρ_{AB}, the expectation value of this joint observable is

$$\langle M_A \otimes N_B \rangle = \mathrm{Tr}[\rho_{AB}(M_A \otimes N_B)]\,.$$

This expression lets us measure correlations.

A basic correlation function is

$$C(M_A, N_B) = \langle M_A \otimes N_B \rangle - \langle M_A \otimes I_B \rangle \langle I_A \otimes N_B \rangle.$$

If

$$C(M_A, N_B) = 0,$$

then the expectation of the product equals the product of the expectations for those observables. If

$$C(M_A, N_B) \neq 0,$$

then the measurement outcomes are correlated.

Correlation functions are central in many-body physics. In later chapters, they will help us understand why Alice's local measurement can reveal information useful for Bob's energy extraction.

3.11 Example: Correlation in a Bell State

Let

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

Let Alice and Bob both measure the Pauli Z observable,

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|.$$

The eigenvalues of Z are +1 for $|0\rangle$ and -1 for $|1\rangle$.

For the Bell state,

$$\langle Z_A \rangle = 0,$$

and

$$\langle Z_B \rangle = 0.$$

But

$$\langle Z_A \otimes Z_B \rangle = 1.$$

So

$$C(Z_A, Z_B) = 1 - 0 \cdot 0 = 1.$$

The individual results are random, but they match perfectly. Alice alone sees randomness. Bob alone sees randomness. Together, they see correlation.

This is precisely the kind of structure that makes entanglement subtle: the information is not stored locally in either subsystem alone. It is stored in the relation between them.

3.12 Bell-Type Correlations

Entanglement is not merely strong correlation. Some correlations produced by entangled quantum states cannot be explained by any theory in which measurement outcomes are predetermined local properties.

This idea began with the Einstein-Podolsky-Rosen argument, which questioned whether the quantum-mechanical description of physical reality was complete (Einstein, Podolsky, and Rosen, 1935). Bell later showed that the issue could be turned into experimentally testable inequalities (Bell, 1964).

The most common undergraduate version is the CHSH inequality, named after Clauser, Horne, Shimony, and Holt (Clauser et al., 1969).

Imagine Alice can choose one of two measurements:

$$A_0, \quad A_1,$$

and Bob can choose one of two measurements:

$$B_0, \quad B_1.$$

Each measurement has possible outcomes +1 or -1.

Define the correlation

$$E(A_i, B_j)$$

as the average value of the product of Alice's and Bob's outcomes when Alice chooses A_i and Bob chooses B_j.

The CHSH combination is

$$S = E(A_0, B_0) + E(A_0, B_1) + E(A_1, B_0) - E(A_1, B_1).$$

Any local hidden-variable theory of the CHSH type satisfies

$$|S| \leq 2.$$

Here, a local hidden-variable theory means a model in which the outcomes are determined by some shared underlying variable, and Alice's choice of measurement cannot instantly affect Bob's local outcome, nor vice versa.

Quantum mechanics predicts that entangled states can violate this bound. For suitable measurements on a maximally entangled pair, quantum theory gives

$$|S| = 2\sqrt{2}.$$

Experiments have observed violations of Bell inequalities, including the time-varying-analyzer experiment of Aspect, Dalibard, and Roger (1982), and many later experiments have refined the tests.

For QET, the lesson is not that Bell violation itself is always required. The lesson is broader: quantum systems can contain nonclassical correlations that are stronger than classical shared-randomness explanations allow.

3.13 Entanglement Does Not Mean Faster-Than-Light Signaling

Bell-type correlations are surprising, but they do not let Alice send a message faster than light.

This is one of the most important points in the whole book.

Suppose Alice and Bob share

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

Bob’s reduced density matrix is

$$\rho_B = \frac{I}{2}.$$

Now Alice measures her qubit in the computational basis.

If Alice obtains 0, Bob’s conditional state becomes

$$|0\rangle\langle 0|.$$

If Alice obtains 1, Bob’s conditional state becomes

$$|1\rangle\langle 1|.$$

But Bob does not know Alice’s outcome unless she sends it to him.

From Bob’s perspective before receiving Alice’s message, he must average over Alice’s possible outcomes:

$$\rho_B' = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{I}{2}.$$

So Bob’s local state is unchanged.

This is the essential no-signaling structure:

Alice’s local measurement can change Bob’s conditional state, but it cannot change Bob’s observable local statistics unless Alice’s outcome is communicated to him.

Quantum information theory formalizes this using completely positive trace-preserving maps, or CPTP maps, which are the general mathematical description of physical quantum operations on density matrices (Nielsen and Chuang, 2010). If Alice applies any local operation and Bob does not learn the outcome, Bob’s reduced state remains unable to carry a controllable message from Alice.

This fact is why QET cannot be used for faster-than-light communication. Bob’s useful operation depends on Alice’s measurement result. Without that classical information, he cannot systematically extract the teleported energy.

3.14 The No-Signaling Calculation

Let the joint state be ρ(AB).

Suppose Alice performs a general local quantum operation with Kraus operators Kμ\ acting only on □(A). The operation is trace-preserving if

$$\sum_\mu K_\mu^\dagger K_\mu = I_A.$$

If Bob does not condition on Alice’s outcome, the new joint state is

$$\rho'_{AB} = \sum_\mu (K_\mu \otimes I_B)\rho_{AB}(K_\mu^\dagger \otimes I_B).$$

Bob’s new reduced state is

$$\rho'_B = \mathrm{Tr}_A(\rho'_{AB}).$$

Substitute:

$$\rho'_B = \mathrm{Tr}_A\left[\sum_\mu (K_\mu \otimes I_B)\rho_{AB}(K_\mu^\dagger \otimes I_B)\right].$$

Using cyclicity inside the partial trace over Alice's subsystem,

$$\rho_B' = \mathrm{Tr}_A\left[\sum_\mu (K_\mu^\dagger K_\mu \otimes I_B)\rho_{AB}\right].$$

Now use trace preservation:

$$\sum_\mu K_\mu^\dagger K_\mu = I_A.$$

Therefore

$$\rho_B' = \mathrm{Tr}_A\left[(I_A \otimes I_B)\rho_{AB}\right] = \mathrm{Tr}_A(\rho_{AB}) = \rho_B.$$

So Alice's unconditioned local operation cannot change Bob's reduced density matrix.

This derivation is one of the mathematical pillars beneath QET. Alice's operation can create conditional changes and correlations. But unless Bob receives the classical outcome, his local statistics remain the same.

3.15 Conditional States: Where Useful Information Appears

No-signaling does not mean Alice's measurement is irrelevant.

It means the information is conditional.

Suppose Alice performs a measurement with outcomes a. Let the measurement operators be M_a, satisfying

$$\sum_a M_a^\dagger M_a = I_A.$$

The probability of outcome a is

$$p(a) = \mathrm{Tr}\left[(M_a \otimes I_B)\rho_{AB}(M_a^\dagger \otimes I_B)\right].$$

If Alice obtains outcome a, the normalized conditional state of Bob is

$$\rho_{B|a} = \frac{\mathrm{Tr}_A\left[(M_a \otimes I_B)\rho_{AB}(M_a^\dagger \otimes I_B)\right]}{p(a)}.$$

Before Bob receives a, he has only

$$\rho_B = \sum_a p(a)\rho_{B|a}.$$

After Bob receives a, he knows which conditional state is relevant.

This is exactly the kind of logic used in QET:

1. Alice measures locally.
2. Her outcome a contains information about the joint state.
3. Bob receives a through a classical channel.
4. Bob chooses an operation depending on a.

The operation is useful only because the state contains correlations between Alice's region and Bob's region.

3.16 Mutual Information: Measuring Total Correlation

We now need a way to quantify correlation.

In classical information theory, mutual information measures how much knowing one random variable reduces uncertainty about another. Quantum theory has an analogous quantity.

First define the von Neumann entropy of a density matrix ρ:

$$S(\rho) = -\,\mathrm{Tr}(\rho \log \rho).$$

If the logarithm is base 2, entropy is measured in bits. If the logarithm is natural, entropy is measured in nats. We will usually state which convention is being used when numerical units matter.

For a bipartite state $\rho_{(AB)}$, the quantum mutual information is

$$I(A : B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

This quantity measures the total correlation between A and B, including both classical and quantum contributions. It is widely used in quantum information theory as a measure of total bipartite correlation (Nielsen and Chuang, 2010).

Let us compare two examples.

3.17 Mutual Information of a Bell Pair

For

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}},$$

the joint state $\rho_{(AB)}$ is pure. Therefore

$$S(\rho_{AB}) = 0.$$

But each reduced state is maximally mixed:

$$\rho_A = \frac{I}{2}, \quad \rho_B = \frac{I}{2}.$$

Each has entropy

$$S(\rho_A) = 1, \quad S(\rho_B) = 1$$

if we use base-2 logarithms.

So

$$I(A : B) = 1 + 1 - 0 = 2$$

bits.

This may look surprising. A Bell pair contains one ebit of entanglement, but its quantum mutual information is two bits. Mutual information counts total correlation in the joint pure state, not only the number of classical bits that can be transmitted.

3.18 Mutual Information of a Classical Correlated Pair

Now consider

$$\rho_{AB}^{\text{classical}} = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|.$$

The reduced states are still

$$\rho_A = \frac{I}{2}, \quad \rho_B = \frac{I}{2},$$

so

$$S(\rho_A) = 1, \quad S(\rho_B) = 1.$$

The joint state has two equally likely alternatives, $|00\rangle$ and $|11\rangle$, so

$$S(\rho_{AB}) = 1.$$

Thus

$$I(A:B) = 1 + 1 - 1 = 1$$

bit.

This matches our intuition: knowing Alice's computational-basis result tells us Bob's computational-basis result. That is one bit of classical correlation.

The comparison is useful:

$$I(A:B)_{\text{Bell}} = 2,$$

while

$$I(A:B)_{\text{classical pair}} = 1.$$

Both states show perfect Z-basis correlation, but the Bell pair contains stronger quantum correlation.

3.19 Correlation Length and Many-Body Systems

So far, our examples used two qubits. QET usually takes place in a many-body system, such as a chain of spins.

Imagine a one-dimensional chain of qubits:

$$1, 2, 3, \dots, N.$$

Alice acts near site 1. Bob acts near site n. The full Hilbert space is

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_N.$$

A local observable near Alice might look like

$$M_A = M_1 \otimes I_2 \otimes I_3 \otimes \cdots \otimes I_N.$$

A local observable near Bob might look like

$$N_B = I_1 \otimes \cdots \otimes I_{n-1} \otimes N_n \otimes I_{n+1} \otimes \cdots \otimes I_N.$$

The correlation between them is

$$C(M_A, N_B) = \langle M_A N_B \rangle - \langle M_A \rangle \langle N_B \rangle.$$

In many physical ground states, correlations tend to decrease with distance. In many gapped one-dimensional systems, connected correlations often decay exponentially with distance, while critical systems can show slower, power-law behavior. The detailed behavior depends on the Hamiltonian and will be studied later when we discuss spin chains.

This matters for QET because the teleported energy depends on correlations between Alice's measurement region and Bob's operation region. If those correlations are very small, the amount of extractable energy is typically small.

3.20 Entanglement as a Resource, But Not a Fuel Tank

It is tempting to say that entanglement is "the energy source" in QET.

That phrase is dangerous.

Entanglement is not a fuel tank. It is not a hidden battery that Bob drains. The actual energy extracted by Bob comes from the local energy of the many-body system after Alice's measurement has disturbed it. Alice's measurement injects energy into the system. The pre-existing correlations in the ground state allow Bob to use Alice's classical information to perform a local operation that lowers the energy near his region.

A better statement is:

Entanglement and correlations are informational resources that make the protocol possible; they are not themselves ordinary stored energy.

This distinction will become precise in later chapters when we introduce Hamiltonians, local energy density, passivity, and energy extraction.

For now, remember:

- Entanglement describes the structure of the quantum state.
- Energy is defined through the Hamiltonian.
- QET uses correlations in the state to guide local energy extraction.
- The classical message tells Bob which operation to apply.
- No useful energy extraction occurs at Bob's side without that classical information.

3.21 Why QET Needs Correlations

Let us connect this chapter directly to QET.

Suppose Alice and Bob's regions are completely uncorrelated. Mathematically, the joint state factors:

$$\rho_{AB} = \rho_A \otimes \rho_B.$$

If Alice performs a measurement, her outcome statistics depend only on $\rho_{(A)}$. Bob's local state remains $\rho_{(B)}$, and Alice's outcome gives no information about Bob's local degrees of freedom.

In that case, Alice's message cannot help Bob choose a better local operation. There is no useful remote information.

But if

$$\rho_{AB} \neq \rho_A \otimes \rho_B,$$

then Alice's outcome may carry information about Bob's conditional state. If the joint system also has a suitable Hamiltonian structure, Bob may use that information to lower the local energy expectation near his region.

This is the conceptual bridge to QET:

$$\text{correlations} \quad + \quad \text{local measurement} \quad + \quad \text{classical communication} \quad + \quad \text{conditi}$$

Each word in that chain matters.

Correlations alone are not enough. Measurement alone is not enough. Classical communication alone is not enough. Bob's operation must be chosen correctly, and the Hamiltonian must permit a decrease in local energy expectation.

3.22 A Simple Preview of the QET Pattern

Consider again the Bell state

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

Alice measures Z. Her outcome tells her whether Bob would obtain 0 or 1 in a Z-measurement. If Alice sends the result to Bob, Bob can condition his action on it.

This is not yet QET, because we have not introduced energy or a Hamiltonian. But structurally,

Document information

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Project	Quantum Energy Teleportation
Document	Document 1.7
Author	mujirin
Verifier	Not verified
Downloaded	July 08, 2026 09:39 KST
Status	Working
Document link	https://theorytrace.com/projects/quantum-energy-teleportation/documents/chapter-3-composite-systems-entanglement-and-correlations/